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A Comparative Study of Applying Active-Set and Interior Point Methods in MPC for Controlling Nonlinear pH Process

Kajian Perbandingan Aplikasi Metode A*ctive-Set* dan *Interior Point* dalam MPC untuk Mengontrol Proses pH Tidak Linear

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Abstract

A comparative study of Model Predictive Control (MPC) using active-set method and interior point methods is proposed as a control technique for highly non-linear pH process. The process is a strong acid-strong base system. A strong acid of hydrochloric acid (HCl) and a strong base of sodium hydroxide (NaOH) with the presence of buffer solution sodium bicarbonate ($NaHCO_3$) are used in a neutralization process flowing into reactor. The non-linear pH neutralization model governed in this process is presented by multi-linear models. Performance of both controllers is studied by evaluating its ability of set-point tracking and disturbance-rejection. Besides, the optimization time is compared between these two methods; both MPC shows the similar performance with no overshoot, offset, and oscillation. However, the conventional active-set method gives a shorter control action time for small scale optimization problem compared to MPC using IPM method for pH control.

Keywords: active-set method, interior point method, model predictive control, pH neutralization process

Abstrak

Studi komparatif Model Predictive Control (MPC) dengan menggunakan metode *active-set* dan metode interior point diusulkan sebagai teknik kontrol untuk proses dengan pH yang sangat non-linear. Proses tersebut merupakan sistem asam kuat-basa kuat. Asam kuat hidroklorida (HCl) dan basa kuat natrium hidroksida (NaOH) dengan kehadiran larutan buffer natrium bikarbonat ($NaHCO_3$) digunakan dalam proses netralisasi yang mengalir menuju reaktor. Model netralisasi pH non-linear yang dibangkitkan dalam proses ini dihadirkan dengan model multi-linear. Kinerja kedua kontroler dikaji dengan mengevaluasi kemampuannya menyelidiki set-point dan penolakan gangguan. Di samping itu, waktu optimasi dibandingkan antara dua metode ini; kedua MPC menunjukkan kinerja yang sama tanpa *overshoot*, *offset*, dan *oscillation*. Bagaimanapun, metode *active-set* konvensional memberikan waktu aksi kontrol yang lebih singkat untuk masalah optimasi skala kecil dibandingkan dengan MPC yang menggunakan metode IPM untuk mengontrol pH.

Kata kunci: metode *active-set*, metode *interior point*, model *predictive control*, proses netralisasi pH

1. Introduction

pH is an indicator for acidity or basicity of an aqueous solution. pH, stands for potential of hydrogen, is also a measure-ment of concentration of hydronium ion (H_3O^+) in a solution. It is also defined as negative logarithm of activity of hydrogen ion activity in a solution.

$$pH = - \log (H^+) \quad (1)$$

pH neutralization process is a very common practice in chemical process, biotechnological industrials and wastewater treatment plants. Some reactions optimally reacts at a certain value of pH (could be off neutral). Soon flowing out from this reactor, the pH must be neutralized for preventing against corrosion. In a waste water treatment plant, the pH of the final discharge has to be maintained around neutral value. In chlorination process of waste water disinfection, the inhabitants' effect of toxicants in wastewater can be assessed and controlled by Ph (Galan *et al*., 2000., Fuente *et al*., 2006., Ruey-Fang, 2004). Furthermore, in textiles and dyeing industries, pH has to be

controlled efficiently because the speed of processing is determined by the pH of the dye bath (Abd El-Rahim *et al*., 2009). In paper industry, pH is important in determining the durability and drying speed of the paper products (Moorthy *et al*., 1998). In metal and mineral industry, extraction process is highly affected by pH. pH has to be controlled to extract the desired metal without dissolving the slag. pH is also important in extracting heavy metals (Chen and Lin, 2001).

Another important application of pH control is in fermentation processes. The increasing understanding about fermentation processes makes the detailed fermentation biochemistry can be applied to design and optimize the process. Out of the many parameters affecting the fermentation, pH is the most important because the pH affects the bacterial growth and pH is the most difficult to be controlled (Menzl *et al*., 1996).

The control of pH is one of the most difficult challenges in the process industry because it shows a strong nonlinear behavior because of the nonlinear characteristics interaction from feed components and ion behavior in reactor. The dynamic of pH process depends on reactants. Different reactants will behave different dynamically. The paper is presented as follows: Modeling of pH strong acid and strong base system and linearization is discussed in section 1. Section 2 reports MPC design technique for both Active-Set and IPM methods. Simulation results are discussed in section 3. It is followed a conclusion remark in section 4.

A MPC controller is designed firstly based on known plant dynamic model including control limitation and measurement limitation. Cost function, stage wise cost and terminal cost, subjected to dynamic motion and limitations, is formed in quadratic programming and solved using either active-set method or interior point method (IPM) in order to obtain the optimum input value, u. The MATLAB function quadprog is used to solve the QP. The methods used to solve the QP can be defined in the algorithms of quadprog function. Tic and Toc MATLAB function is used to calculate the action time in solving the QP problem.

In MPC, there are three types of model that can be used for the controller design. The first one is finite impulse response model (FIR)/step response model. This type of model is preferred because process time delay, response time and gain are included. However, this model can only be applied to stable plant. A transfer function model can describe stable and unstable plant well, but the downside is its ineffectiveness in handling multivariable plants. The third model is state-space model which can handle both stable and unstable plant with multivariables (Wang, 2009).

Figure 1 shows schematic diagram of MPC control system. Input, u, is introduced into the plant in which the linearized pH model is utilized to calculate the output, pH. The pH value obtained is fed into MPC controller. Inside the controller block, state variables, output variables and set-point are introduced into the algorithm of MPC to calculate the next input variables across the prediction horizon. However, only the first input variable is chosen to feedback into the plant. Then the plant produces the next output variable.

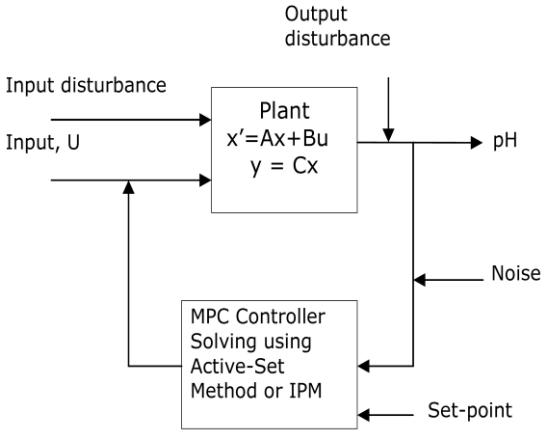

Figure 1. Schematic representation of MPC control system

Disturbances are introduced into the system to test the controller's ability to reject disturbance. Three types of disturbances are considered. The first is the input disturbance, which is caused by inconsistent acid flow into the system. Output disturbance and noise may also be found along the way from plant to the controller.

In this study, state-space function is used. For a single-input and single-output (SISO) system,

$$x_m(k+1) = A_m x_m(k) + B_m u(k), \quad (2)$$

$$y(k) = C_m x_m(k) + D_m u(k), \quad (3)$$

where u is the manipulated variable or input variable; y is the process output; and x_m is the state variable vector with assumed dimension n_1. However, we have to implicitly assume that input signal $u(k)$ cannot

affect the output $y(k)$ because of the receding horizon control where current information of the plant is needed for prediction and control. As a result, D_m is assumed to be zero.

$$y(k) = C_m x_m(k) \tag{4}$$

Let denotes the difference of state variables and difference of control variables as followed:

$$\Delta x_m(k+1) = x_m(k+1) - x_m(k);\ \Delta x_m(k) = x_m(k) - x_m(k-1), \tag{5}$$

$$\Delta u(k) = u(k) - u(k-1) \tag{6}$$

Substitute equations 5 and 6 into equation 4, the difference of the state-space equation is:

$$\Delta x_m(k+1) = A_m \Delta x_m + B_m \Delta u(k) \tag{7}$$

A new state variable vector is chosen to connect Δx_m and y

$$x(k) = \begin{bmatrix} \Delta x_m(k) & y(k) \end{bmatrix}^T \tag{8}$$

where superscript T indicates matrix transpose. For output variable, it is

$$\begin{aligned} y(k+1) - y(k) &= C_m(x_m(k+1) - x_m(k)) \\ = C_m \Delta x_m(k+1) &= C_m A_m \Delta x_m(k) + C_m B_m \Delta u(k) \end{aligned} \tag{9}$$

Putting together equations 7 and 9,

$$\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k), \tag{10}$$

$$y(k) = \begin{bmatrix} 0_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \tag{11}$$

where 0_m = [0 0 …. 0].

Equations 10 & 11 can be denoted as:

$$x(k+1) = Ax(k) + B\Delta u(k) \tag{12}$$

$$y(k) = Cx(k) \tag{13}$$

where *A, B, C* are called the augmented model which is used in the predictive control. Assuming that state variables are measurable, the future control trajectory is denoted by

$$\Delta u(k_i),\ \Delta u(k_i+1), \cdots, \Delta u(k_i+N_c-1) \tag{14}$$

where N_c is the control horizon to capture the future control trajectory. With this information, future state variables are predicted.

$$x(k_i+1 \mid k_i),\ x(k_i+2 \mid k_i), \cdots, x(k_i+m \mid k_i), \cdots, x(k_i+N_p \mid k_i)$$

where N_P is the prediction horizon.
Given that:

$$Y = Fx(k_i) + \Phi \Delta u \tag{15}$$

where

$$Y = \begin{bmatrix} y(k_i+1 \mid k_i) & y(k_i+2 \mid k_i) & \cdots & y(k_i+Np \mid k_i) \end{bmatrix}^T$$

$$\Delta u = \begin{bmatrix} \Delta u(k_i) & \Delta u(k_i+1) & \cdots & \Delta u(k_i+Nc-1) \end{bmatrix}^T$$

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{Np} \end{bmatrix}$$

and

$$\Phi = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{Np-1}B & CA^{Np-2}B & CA^{Np-3}B & \cdots & CA^{Np-Nc}B \end{bmatrix}$$

1.1. Quadratic Programming

In algorithm of MPC, the objective is to bring the predicted output to the set-point signal $r(k_i)$. In order to do so, a control parameter vector ΔU is calculated by minimizing the cost function (J) which is defined as:

$$J = (R_s - Y)^T Q (R_s - Y) + \Delta U^T R \Delta U, \tag{16}$$

where R_s contain the set-point information, Q and R are tuning parameters for the closed loop performance. One of the benefits of MPC is its ability to handle constraint. There are three main types of constraints that are usually encountered in control application, which are constraint on amplitude of control variables, U, rate of change of control variable ΔU, and amplitude of output Y.

$$\Delta u^{min} \le \Delta u(k) \le \Delta u^{max} \tag{17}$$

$$u^{min} \le u(k) \le u^{max} \tag{18}$$

$$y^{min} \le y(k) \le y^{max} \tag{19}$$

Combining all these constraints in a compact matrix,

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} \Delta U \le \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \quad (20)$$

where the data matrices are

$$M_1 = \begin{bmatrix} -C_2 \\ C_1 \end{bmatrix}; N_1 = \begin{bmatrix} -u^{\min} + C_1 u(k_i - 1) \\ u^{\min} - C_1 u(k_i - 1) \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -I \\ I \end{bmatrix}; N_2 = \begin{bmatrix} -\Delta u^{\max} \\ \Delta u^{\max} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} -\Phi \\ \Phi \end{bmatrix}; N_3 = \begin{bmatrix} -y^{\min} + Fx(k_i) \\ y^{\min} - Fx(k_i) \end{bmatrix}$$

This compact matrix consists of three types of constraints where M_1, N_1, refer to constraint on amplitude of control variables; M_2, N_2, refer to constraint on rate of change of control variables; M_3, N_3, refer to constraint on amplitude of output. In short, the control action is to minimize the cost function J subjected to constraints $Mx \le \gamma$.

The cost function to be minimized in MPC algorithm has the form of standard quadratic programming (QP) problem (Wang, 2009). This QP problem has been extensively studied in literature and several methods have been proposed to solve this QP problem. The standard quadratic program-ming has the form:

$$J = \tfrac{1}{2} x^T E x + x^T F \quad (21)$$

$$Mx \le \gamma \quad (22)$$

where E, F, M and γ are compatible matrices and vectors in QP. E has to be symmetric and positive definite. Two methods of solving QP are of interest, which are active-set method and IPM.

1.2. Active-set Method

The way of solving QP with active set method is to define a set of constraints to be active set at each step of algorithm. This working set is a subset of constraint that is actually active at current point. Next, the algorithm moves to an improved point on the surface defined by the previous subset of constraint. Thus, at each step, QP with equality constraints is solved until all Lagrange multipliers are positive to obtain the local solution. In order to minimize the cost function subjected to equality constraint, Lagrange expression is considered as

$$J = \tfrac{1}{2} x^T E x + x^T F + \lambda^T (Mx - \gamma) \quad (23)$$

By taking the partial derivative of this Lag-range expression to x and γ and equal to zero to obtain:

$$\lambda = -(ME^{-1}M^T)^{-1}(\gamma + ME^{-1}F) \quad (24)$$

$$x = -E^{-1}F - E^{-1}M^T\lambda \quad (25)$$

1.3. Interior Point Method (IPM)

As mentioned before, QP can be solved in many ways; IPM is one of the well-developed methods. IPM also works well in many applications. IPM uses iteration to solve QP and always guarantee that each iteration satisfies the constraints in strict manner. The solution will never lies on the constraint. In order to solve QP with inequality constraints, Karush-Kuhn-Tucker condition is introduced. Active and inactive con-straints are now defined as Lagrange multiplier. Newton's method is used to obtain the solution. The conditions are as following

$$Ex + F - M\lambda = 0 \quad (26)$$

$$M^T x - \gamma \ge 0 \quad (27)$$

$$\lambda(M^T x - \gamma) = 0 \quad (28)$$

$$\lambda \ge 0 \quad (29)$$

Function, F, is defined such that the roots of this function are the optimum solution of the conditions. Residual vectors, r_d, r_p, r_s are also defined in this function.

$$\mathcal{F} = \begin{bmatrix} Ex - M\lambda + F \\ s - A^T x + b \\ s \prod e \end{bmatrix} = \begin{bmatrix} r_d \\ r_p \\ r_s \end{bmatrix} \quad (30)$$

where $s = M^T x - \gamma$, the slack vector and $e = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ is to be solved. A Jacobian matrix is used to obtain the search direction.

$$J(x,\lambda,s)\begin{bmatrix}\Delta x\\ \Delta\lambda\\ \Delta s\end{bmatrix} = -\begin{bmatrix}r_d\\ r_p\\ r_s\end{bmatrix} \quad (31)$$

where $J(x,\lambda,s) = \begin{bmatrix} E & -M & 0\\ -M^T & 0 & I\\ 0 & S & \Pi\end{bmatrix}$

The next iterates is calculated including a line search parameter *a*.

$$(x_{k+1},\lambda_{k+1},s_{k+1}) = (x_k,\lambda_k,s_k) + \alpha(\Delta x,\Delta\lambda,\Delta s) \quad (32)$$

Predictor and corrector method is involved in IPM algorithm. The predictor step will determine the value of centering parameter using complementary measure, μ

$$\mu = \frac{s^T\lambda}{m} \quad (33)$$

$$\mu^{aff} = \frac{\left(s+\alpha^{aff}\Delta s^{aff}\right)^T\left(\lambda+\alpha^{aff}\Delta s^{aff}\right)}{m} \quad (34)$$

$$\sigma = \left(\frac{\mu^{aff}}{\mu}\right)^3 \quad (35)$$

Last but not least, a corrector step is used instead to obtain the search direction so that iterates proceed in the center path towards the solution.

$$\begin{bmatrix} E & -M & 0\\ -M^T & 0 & I\\ 0 & S & \Pi\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta\lambda\\ \Delta s\end{bmatrix} = \begin{bmatrix}r_d\\ r_p\\ r_s + r_t\end{bmatrix} \quad (36)$$

$$r_t = \Delta s^{aff}\Delta\Pi^{aff}\,e - \sigma\mu e \quad (37)$$

New values of x,λ,s are updated and the iterations continue until a solution is obtained.

2. Methods

Modeling of pH processes has been started on 1972 by McAvoy (McAvoy *et al*., 1972) based on mass balance and chemical equilibrium components as the base of mathematical model. As an extention to that, Gustafsson and Waller (1983) proposed the reaction invariant concept (Gustafsson and Waller, 1983) in which an adaptive pH system was designed with the concentration of weak acid as adjustable parameters (Nejati *et al*., 2012). The adaptive system has succesfully dealt with time varying properties; however, prior information on the feed stream is required (Sung *et al*., 1998). Wright and Kravaris 1991 proposed a reduced form of pH process model in first order state equation. A strong acid equivalent was introduced (Wright and Kravaris, 1991). The two con-ditions required for this state equation to hold are that the initial state is in steady state and the feed composition does not change.

In this study, a pH process system uses Sodium hydroxide (NaOH) to neutralize hydro cloride acid (HCl) stream with the present of $NaHCO_3$ in a continuous stirred reactor. The process is shown in Figure 2.

2.1. pH Process Modeling

A pH system is considered as proposed by Galan *et al*. (2000) which is a continuous stirred tank reactor (CSTR) with a constant volume. A strong base with volumetric flowrate q_B (t) of composition base ($x_{2,i}$) and buffer agent ($x_{3,i}$) is to neutralize acid solution with flowrate $q_A(t)$ with acid composition $x_{1,i}$. The dynamic models are as follows:

$$\dot{x}_1 = \tfrac{1}{\theta}\left(x_{1,i} - x_1\right) - \tfrac{1}{\theta}x_1 u \quad (38)$$

$$\dot{x}_2 = -\tfrac{1}{\theta}x_2 + \tfrac{1}{\theta}\left(x_{2,i} - x_2\right)u \quad (39)$$

$$\dot{x}_3 = -\tfrac{1}{\theta}x_3 + \tfrac{1}{\theta}\left(x_{3,i} - x_3\right)u \quad (40)$$

$$h(x,y) = \xi + x_2 + x_3 - x_1 - \frac{K_w}{\xi} - \frac{x_3}{1+\frac{K_x\xi}{K_w}} = 0, \quad (41)$$

where ξ is the concentration of H^+ ions, $\theta = \frac{v}{q_A}$, $u = \frac{q_B}{q_A}$.

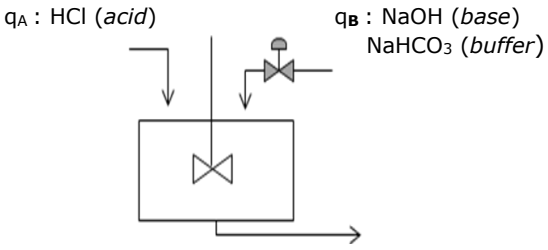

Figure 2. pH neutralization process

A pH process system of NaOH is used to neutralize HCl stream with the presence of $NaHCO_3$ in a continuous stired reactor. The behavior is shown in Figure 3. As shown in the Figure 3, the titration curve is divided and represented by 5 linear models. They are reported in Galan *et al*. (2000).

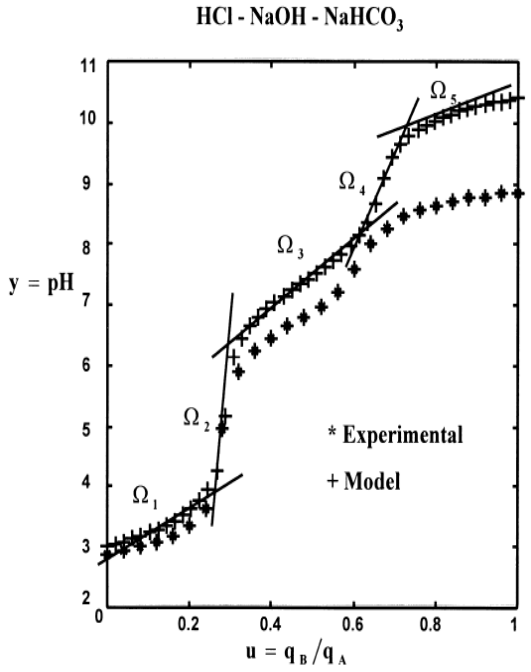

Figure 3. Titration curve (input-output map) for the system and multi-linear model identification. The comparison shows between the experimental and the theoretical values (Galan *et al*., 2000)

2.2. Linearize Model

The pH model used in this research is used as proposed by Galan *et al*. (2000). The linearized mass balance equation is described in a general state-space model:

$$A = \begin{bmatrix} -\frac{1}{\theta}(1+u_s) & 0 & 0 \\ 0 & -\frac{1}{\theta}(1+u_s) & 0 \\ 0 & 0 & -\frac{1}{\theta}(1+u_s) \end{bmatrix} \quad (42)$$

$$B = \begin{bmatrix} -\frac{1}{\theta}x_{1,s} \\ \frac{1}{\theta}(x_{2,i}-x_{2,s}) \\ \frac{1}{\theta}(x_{3,i}-x_{3,s}) \end{bmatrix} \quad (43)$$

$$C = \left[\frac{\delta\eta}{\delta x_1} \quad \frac{\delta\eta}{\delta x_2} \quad \frac{\delta\eta}{\delta x_3}\right]^T \quad (44)$$

where

$$\frac{\delta\eta}{\delta x_k} = \frac{\delta h / \delta x_k}{\xi \ln(10)\, \delta h / \delta \xi}, \quad k = 1,2,3$$

Where

$$\frac{\delta h}{\delta \xi} = 3K_x\xi^2 + 2\left[K_w + (x_3 + x_2 - x_1)K_x\right]\xi + (x_2 - x_1 - K_x)K_w$$

$$\frac{dh}{dx_1} = -K_x\xi^2 - K_w\xi$$

$$\frac{dh}{dx_2} = K_x\xi^2 + K_w\xi$$

and

$$\frac{dh}{dx_3} = K_x\xi^2.$$

3. Results and Discussion

For set-point tracking study, the set-point is prescribed within the range of pH 6.4 to 8.3. For this simulation, the pH set-points are pH 7 at the first 20 minutes, pH 7.5 for the next 20 minutes, followed by pH 8, pH 7 and pH 6.5. For the disturbance rejection study, the pH set-point is set to be pH 7 throughout the simulation while the acid input, q_A is set as measurable disturbance by varying its value periodically from 1 L/min to 1.4 L/min and 0.6 L/min.

During the simulation, the state variables are assumed to be measurable. The flow rate of the acid stream is assumed to be constant at 1 L/min. The linear model that is used for this simulation is in the range of pH 6.4 to 8.3, which is an approximation linear model at the neutralization point, pH 7. The tuning parameters used for this MPC con-controller are as follows (Table 1).

Table 1. Design parameters of MPC controller

Parameters	Values
Prediction horizon, NP	20
Control horizon, NC	4
Weighting factor, Q	50I
Weighting factor, R	0.02I

As mentioned before, the linear model used is only applicable to the certain range of pH, thus the output constraint is set to be in between pH 6.4 and pH 8.3. Besides, the constraint of the input stream, which is the titrating stream, is constrained in the range of 0 - 1000 mL/min. Another constraint that is applied in this simulation is on the rate of change of the input stream, Δu. The reason to apply this constraint is that the rate of change of input stream is restricted by the type of control valve. The actuator delays in opening or closing the valve in order to reach the desired set-point.

3.1. Simulation Result for MPC Controller using IPM Method (Set-Point Tracking)

The simulation result for MPC using IPM is shown in Figure 4. The set-point tracking performance of the controller is considered

good. The output tracks smoothly the set-point without offset and overshoot. The set-point is set to be within the range of the allowable pH range of the linear model. The time required for the controller to track the set-point is around one minute. The constraints applied for this controller are output disturbance and input disturbance. From Figure 4, the dotted lines are the illustration of the constraint.

In addition, output disturbances and noises on feedback states are also included during this simulation to observe what the response of the controller towards these disturbances and noises. The noise and the output disturbance used in the simulation are random number with 0.02 weights. From Figure 4, the pH does not deviate much from the output disturbance and noise and is able to return to the set-point quite fast.

As shown in Figure 5, the output response to the changes in set-point is slower if constraint on rate of change of input stream is small. In this simulation, the constraint on rate of change of input is $-0.1<\Delta u<0.1$. In other words, the rate of change of input is only allowed to be 0.1 L/min. It requires 2 minutes to change from pH 7.5 to pH 8 at sampling time 40 minutes. On the other hand, for larger changes in pH set-point which we can observe at sampling time 60 minutes, the controller consumes around 3 minutes to change from pH 8 to pH 6. Comparing the control action of both cases, control action of Figure 4 is more exaggerate than that of Figure 5 because there is no constraint active on the rate of change on the input for the first case. In addition, the dots that represent the constraints on the output, input and rate of change of input are shown clearly in Figures 4 and 5. One of the advantages of MPC is that constraints can be introduced directly into the MPC algorithm. In this case, constraints are as followed:

$6.4 < \text{pH} < 8.2$,
$0 < u < 1.0$,
$-0.1 < \Delta u < 0.1$, or $-1 < \Delta u < 1$.

From Figure 4, it shows that all three types of constraints are not active of violated. Output pH, input u, and increment Δu work within the range of the constraints. However, in Figure 5, out of those three constraints, only the constraints on Δu are active. This is the reason why the pH responses slowly to change set-point compared to that of Figure 4.

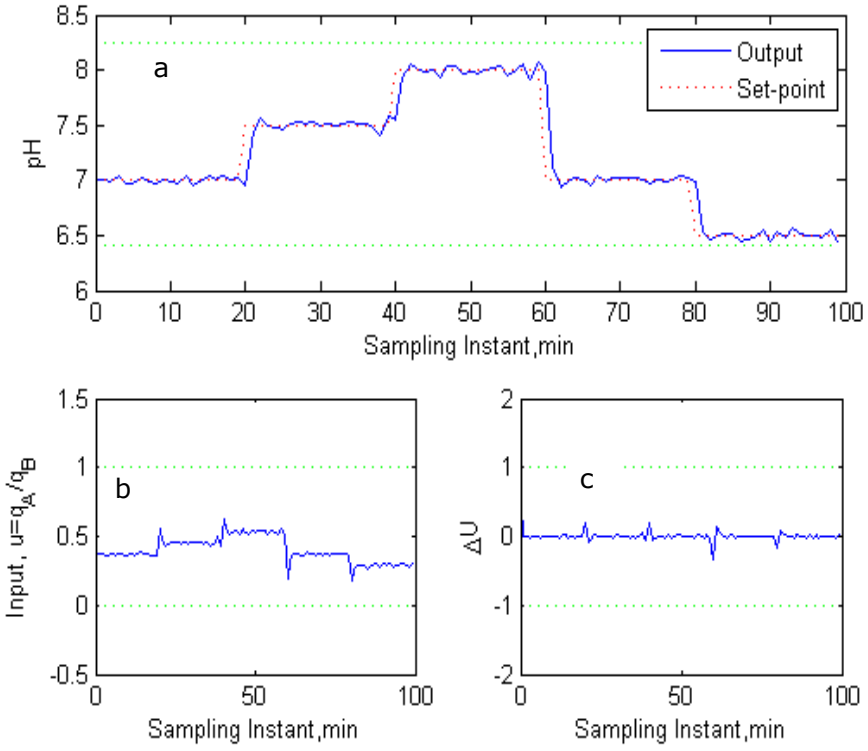

Figure 4. Output response (a), manipulated variable response (b), incremental input (c) for set-point tracking using MPC

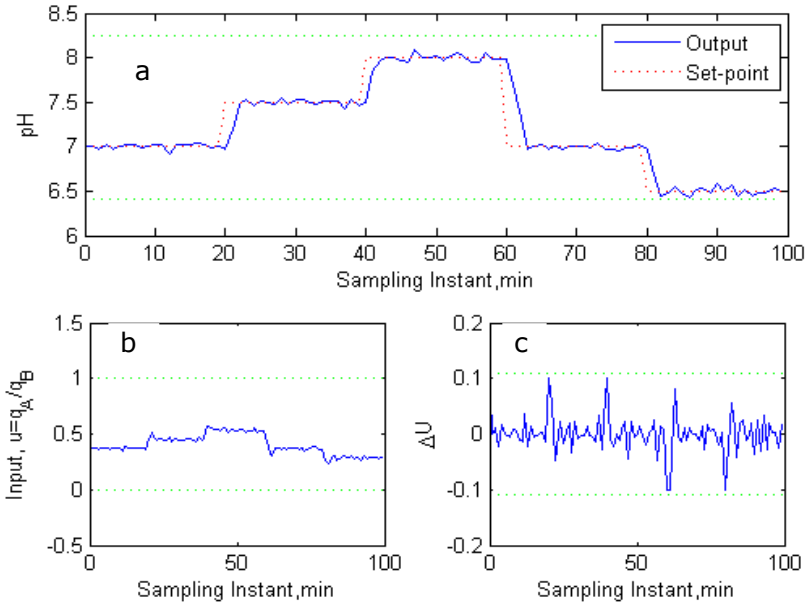

Figure 5. Output response (a), manipulated variable response (b), incremental input (c) for set-point tracking using MPC of IPM with constraint on Δ*u*

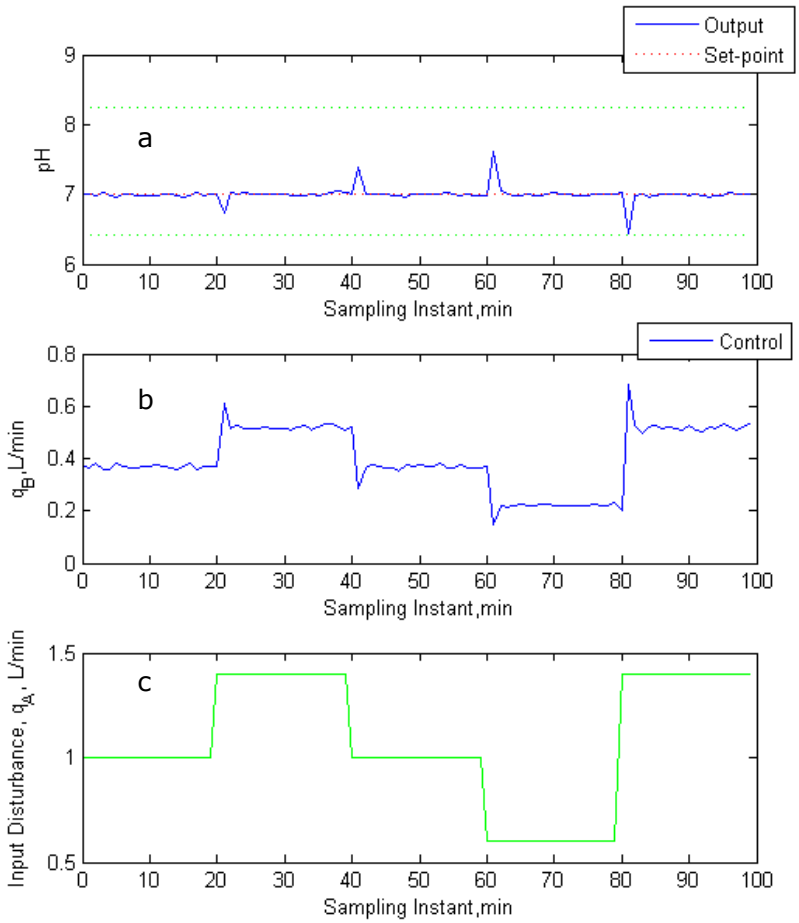

Figure 6. Output response (a), manipulated variable response (b), for distur-bance in acid flow (c) using MPC with IPM

3.2. Simulation Result for MPC Controller using IPM Method (Disturbance Rejection)

The next simulation is disturbance rejection. The purpose of this simulation is to check the performance of the controller in case that measurable disturbance is injected into the system. This is an important test on a controller to check whether the controller is able to reject disturbance without further deviation from the set-point. In this simulation, the acid stream is changed from time to time unlike the previous simulation where the acid stream is assumed to be constant all the time at 1 L/min. The changes in flow rate of acid stream can be observed in Figure 6(c). Besides, the set-point of pH is remained at pH 7 throughout the simulation. As shown in Figure 6, when the flow rate of acid stream is increased, the controller opens the valve of titrating stream to counter-act with the decrease of pH and vice-versa. The output response is quite good as the output response is able to return to the set-point without much delay and deviation from the set-point. The time required for the output to return back to the set-point is around 2 minutes as shown in Figure 6.

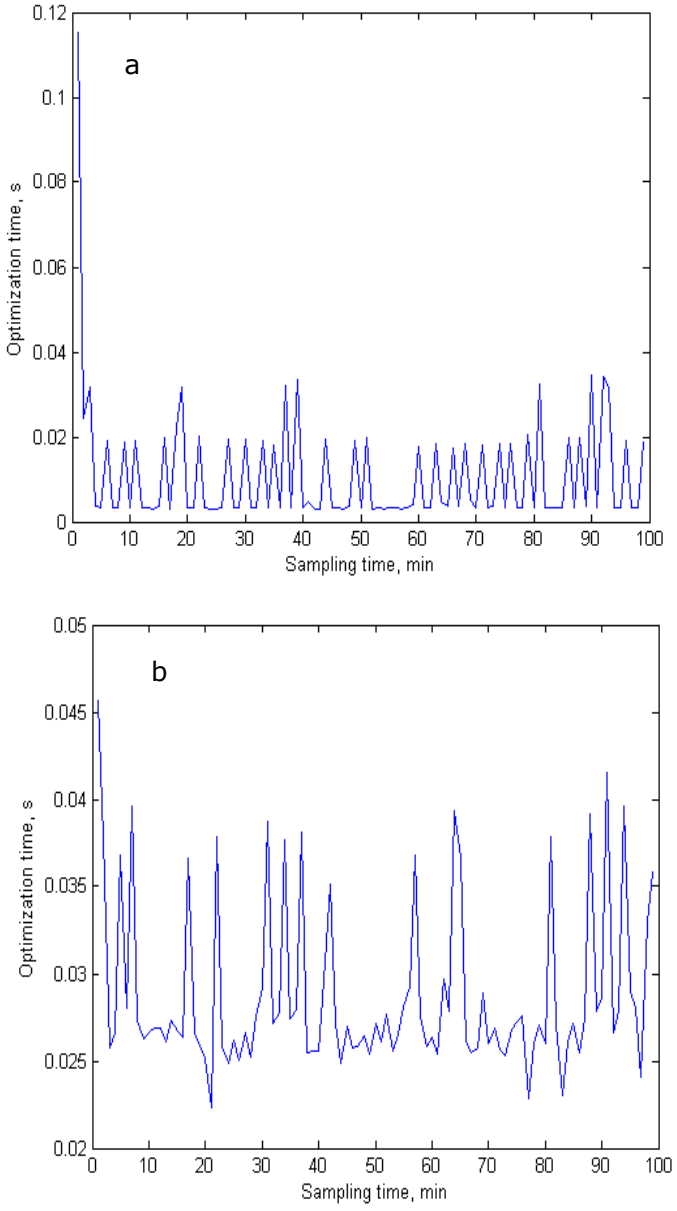

Figure 7. Control action time of MPC using active set method (a), and IPM (b)

3.3. Comparison of Optimization Time

It is interesting to compare the optimization action time of MPC that uses IPM with that of using active set method. IPM is expected to be a faster method in solving a quadratic programming (QP) problem. For active set method, a set of constraint is treated as active set at each step of the algorithm. At each step, an equality constraint problem is solved. Then the algorithm proceeds to another set of constraint until all Lagrange multipliers are equal or greater than zero. If there are many constraints to be solved, the computational load will be large and the action time will be longer. While IPM solves the QP by finding the optimum solutions using decision variables within the feasible region rather than solving each constraint one by one. In this simulation, tictoc function is used to calculate the optimization time of solving the quadratic programming problem within the MPC algorithm. From Figure 7 (a), it is shown that the optimization time for MPC using active set method are within the range of 0.01 to 0.05s. The average action time is 0.0191 s. It is shown in Table 2.

On the other hand, the optimization time of MPC using IPM throughout the simulation are recorded and presented in Figure 7 (b). The optimization time is in the range of 0.03 to 0.06 s while the mean control action time is 0.0464 s. From this result, it shows that active set method is faster than IPM in solving the QP problem in MPC for pH neutralization. Although IPM is expected to solve QP much faster than active set method because active set method solves each constraint one by one to obtain the optimum solution. However in this particular case, pH neutralllization, each MPC step requires solution of QP with 60 variables and constraints only since this is a single input system with 3 states and prediction horizon of 20 points.

Table 2. Comparison of optimization time between IPM and active-set method

Methods	Optimization time, s
IPM	0.0191
Active-Set method	0.0354

4. Conclusion

A model predictive controller using active set and interior point methods is designed for highly non-linear pH control. The parameter for the designed MPC controller are N_P=20, N_C=4, Q=50I, R=0.02I. Performance of the controller is simulated on set-point tracking and disturbance rejection aspect. The simulation result shows that MPC with IPM offers good set-point tracking and disturbance rejection without offset, overshoot, and undershoot. Besides, a comparison between methods of solving quadratic programming problem exists in MPC algorithm, which is IPM and active set method. The simulation result shows that MPC with active set method is faster than IPM for this particular pH control problem because pH control system is not complex and do not involved enormous amount of constraints. It can be concluded that IPM method would be faster in a more complex system.

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